SIMULTAION OF THE THERMAL CONDITIONS OF A FRACTURED GEOTHERMAL RESERVOIR

S. A. Safonov

UDC 532.546

The model proposed here for heat exchange in an undeformed fractured geothermal reservoir takes into account the thermal resistance of the rock slabs constituting a stratum and heat exchange with the massif surrounding the stratum.

The thermal conditions of a vertical geothermal fractured stratum under the conditions of nonisothermal filtering without allowance for the thermal resistance of the rock slabs was studied in [1]. A semi-analytic method of taking the heat loss in rock slabs into account was suggested in [2]. It was based on the assumption that the temperature in a rock slab along the normal to its surface varies as $T_b(n, t) = (a + bn + cn^2)\exp(-n/\sqrt{a_b t})$ (n is the normal and a_b is the thermal diffusivity of the slab).

We consider the problem of determining the thermal conditions of a fractured horizontal geothermal stratum under the conditions of nonisothermal filtering in the following formulation. We introduce the x, y, z coordinate system so that the plane z = 0 would coincide with the top of the stratum. The thickness h of the stratum is assumed to be much less than that of the cap rock. In this case the problem is symmetric about the plane z = h/2. We assume that:

1) the thickness of the stratum is much smaller than its dimensions in the xy plane and the filtering rate field is two-dimensional, i.e., there is no component of the rate along the z axis, perpendicular to the stratum;

2) the heat exchange at the solid–liquid interface is so intense (Biot number $Bi = \alpha L_b/\lambda_b >> 1$, α is the coefficient of interphase heat transfer, L_b is the characteristic size of the rock slab, and λ_b is the thermal conductivity of the slab material) that the temperature of the phases at the interface become equal almost instantaneously in comparison with the characteristic useful life of the stratum;

3) the slabs of rock that constitute the stratum are in the form of regularly arranged parallelepipeds;

4) conductive heat transfer in the massif enclosing the stratum in the direction of the filtering can be ignored; and

5) the stratum is undeformable.

The analysis of the problem is based on the equation of the heat balance of the filtering liquid:

$$mc_w \frac{\partial T}{\partial t} + c_w \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = Q_b + m\lambda_w \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right). \quad (1)$$

Here T is the temperature of the liquid; u and v are the components of the filtering rate; Q_b is the source term responsible for the heat exchange of the liquid with the rock slabs; m is the porosity (fracturing); and λ_w and c_w are the thermal conductivity and volume heat capacity of the liquid.

Averaging (1) over z from 0 to h/2, we obtain

$$mc_w \frac{\partial \overline{T}}{\partial t} + c_w \left(u \frac{\partial \overline{T}}{\partial x} + v \frac{\partial \overline{T}}{\partial y}\right) = \overline{Q}_b + m\lambda_w \left(\frac{2}{h} \frac{\partial T}{\partial z}\Big|_{z=0} + \frac{\partial^2 \overline{T}}{\partial x^2} + \frac{\partial^2 \overline{T}}{\partial y^2}\right) \quad (2)$$

Novosibirsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 5, pp. 127-132, September-October, 1993. Original article submitted May 6, 1992; revision submitted October 15, 1992.

 0021-8944/93/3405-0716$12.50

(the bar pertains to the average value of the quantity and is omitted henceforth).

The motion of the liquid is described by the Darcy law

$$u = -\frac{k_e}{\mu(T)}\frac{\partial p}{\partial x}, \quad v = -\frac{k_e}{\mu(T)}\frac{\partial p}{\partial y}, \tag{3}$$

where p is the pressure; k_e is the effective permeability of the isotropic stratum ([3]); and $\mu(T)$ is the viscosity of the liquid. According to [4]

$$\mu(T) = 241\cdot 10^{\frac{248}{133+T}-7}\ \text{N}\cdot\text{sec/m}^2.$$

The mass balance equation for a stratum opened by two wells with equal flow rates, one an extraction and a water injection well, has the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{Q}{h}\left(\delta(x - x_{s0}, y - y_{s0}) - \delta(x - x_{si}, y - y_{si})\right). \tag{4}$$

Here x_{s0}, y_{s0} and s_{xi}, y_{si} are the coordinates of the two-dimensional source and drain simulating the wells; Q is the well flow rate; and δ is the delta function.

Boundary conditions of the fourth kind are satisfied for z = 0:

$$T_r = T, \quad \lambda_r\frac{\partial T_r}{\partial z} = m\lambda_w\frac{\partial T}{\partial z}$$

(T_r is the temperature of the enclosing massif; and λ_r is the thermal conductivity).

The heat flow from the massif is determined from the heat conduction equation

$$\frac{\partial T_r}{\partial t} = a_r\frac{\partial^2 T_r}{\partial z^2} \tag{5}$$

(a_r is the thermal diffusivity) with the boundary conditions

$$T_r(z, 0) = T_0, \quad T_r(0, t) = T(x, y, t).$$

The source term in Eq. (2), averaged over the thickness of the stratum, is determined from the solution of the conjugate problem of heat conduction

$$\frac{\partial T_b}{\partial t} = a_b\left(\frac{\partial^2 T_b}{\partial \xi^2} + \frac{\partial^2 T_b}{\partial \eta^2} + \frac{\partial^2 T_b}{\partial \zeta^2}\right), \tag{6}$$

$$-a/2 < \xi < a/2, \quad -b/2 < \eta < b/2, \quad -c/2 < \zeta < c/2, \quad T_b(\xi, \eta, \zeta, 0) = T_0,$$

$$T_b(\pm a/2, \eta, \zeta, t) = T_b(\xi, \pm b/2, \zeta, t) = T_b(\xi, \eta, \pm c/2, t) = T(x, y, t)$$

(T_b is the temperature in the slab of rock).

At an infinite distance from the wells the temperature of the liquid is T_0 during the entire filtering process and the filtering rate is zero. We assume that the temperature of the injected liquid $T(x_{si}, y_{si}, t) = T_s$ is established instantaneously at the boundary of the injection well.

The solution of the boundary-value problem (6) has the form [5]

$$T_b(\xi, \eta, \zeta, t) = T_0 f(\xi, \eta, \zeta, t) + \frac{\partial}{\partial t}\int_0^t T_b(\xi, \eta, \zeta, t-\chi)\,[1 - f(\xi, \eta, \zeta, \chi)]\,d\chi, \tag{7}$$

$$f(\xi, \eta, \zeta, \chi) = 8\sum_{m,n,k=1}^{\infty}\frac{(-1)^{m+n+k+1}}{\mu_m\mu_n\mu_k}\times$$

$$\times\cos\left(\mu_n\frac{2\xi}{a}\right)\cos\left(\mu_m\frac{2\eta}{b}\right)\cos\left(\mu_k\frac{2\zeta}{c}\right)e^{-\chi\beta_{nmk}},$$

$$\mu_l = (2l - 1)\pi/2, \quad l = n, m, k, \quad \beta_{nmk} = 4a_b\left[(\mu_n/a)^2 + (\mu_m/b)^2 + (\mu_k/c)^2\right].$$

Equation (7) indicates that the temperature gradient at the rock slab boundary $\xi = a/2$ for slow heat-transfer processes is

$$\frac{\partial T_b}{\partial \xi}(\cdot, t)\Big|_{\xi=a/2} \cong T_0 \frac{\partial f}{\partial \xi}(\cdot, t)\Big|_{\xi=a/2} - \frac{\partial}{\partial t}\int_0^t \left[T(\cdot, t) - \chi \frac{\partial T}{\partial t}(\cdot, t)\right] \frac{\partial f}{\partial \xi}(\cdot, \chi)\Big|_{\xi=a/2} d\chi,$$

and the total heat flux across the indicated facet of the rock slab, reduced to a unit volume of slab (we omit the intermediate manipulations), is

$$q_a(t) = -\frac{1}{abc}\int\int \frac{\partial T_b}{\partial \xi}(a/2, \eta, \zeta, t)\, d\eta d\zeta \cong \frac{16}{a^2}\left\{-T_0 \sum_{n,m,k=1}^{\infty} \frac{e^{-\beta_{nmk}t}}{\mu_m^2\mu_k^2} + \right. \tag{8}$$

$$\left. + \sum_{n,m,k=1}^{\infty} \frac{1}{\mu_m^2\mu_k^2} \frac{\partial}{\partial t}\left\{T \frac{1 - e^{-\beta_{nmk}t}}{\beta_{nmk}} - \frac{\partial T}{\partial t}\left[\frac{1 - e^{-\beta_{nmk}t}}{\beta_{nmk}^2} - \frac{te^{-\beta_{nmk}t}}{\beta_{nmk}}\right]\right\}\right\}.$$

Differentiating the expression after the second summation sign in (8) with respect to t and disregarding the second derivative of the temperature with respect to time because of the assumption of a slow heat transfer, we finally have

$$q_a(t) \cong \frac{16}{a^2} \sum_{n,m,k=1}^{\infty} \frac{1}{\mu_m^2\mu_k^2}\left[(T - T_0)\, e^{-\beta_{nmk}t} - \frac{\partial T}{\partial t} \frac{1 - e^{-\beta_{nmk}t}(1 + \beta_{nmk}t)}{\beta_{nmk}}\right].$$

The total heat flux across the surface of the rock slab ($q_a(t)$ and $q_b(t)$ are the heat fluxes across the boundaries $\eta = b/2$ and $\zeta = c/2$, respectively)

$$Q(t) = 2\,(q_a(t) + q_b(t) + q_c(t)). \tag{9}$$

The heat flux across the boundary between the enclosing massif and the stratum is determined from the solution of Eq. (5) with given boundary conditions:

$$q_r = -\frac{2\lambda_r}{h} \frac{\partial T_r}{\partial z}\Big|_{z=0} = -\frac{2\sqrt{\lambda_r c_r}}{h} \frac{d}{dt}\int_0^t \frac{T(x, y, \tau) - T_0}{\sqrt{\pi(t - \tau)}} d\tau.$$

For a flow heat transfer between the enclosing massif and the stratum we obtain [6]

$$q_r \cong \frac{2}{h}\sqrt{\frac{\lambda_r c_r}{\pi}}\left(\frac{T - T_0}{t^{0.5}} + \frac{\partial T}{\partial t} t^{0.5}\right) \tag{10}$$

(c_r is the volume heat capacity of the material of the enclosing massif).

The system of equations (2)-(4) with allowance for (9) and (10) in dimensionless variables is reduced to

$$c(t)\frac{\partial \theta}{\partial t} + \frac{\partial (u\theta)}{\partial x} + \frac{\partial (v\theta)}{\partial y} = (1 - \theta)\, d(t) + m\mathrm{Pe}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + \tag{11}$$

$$+ \theta\,(\delta(x - x_{s0}, y - y_{s0}) - \delta(x - x_{si}, y - y_{si})),$$

$$\frac{\partial}{\partial x}\left(\bar{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\bar{\mu}\frac{\partial p}{\partial y}\right) = \delta(x - x_{si}, y - y_{si}) - \delta(x - x_{s0}, y - y_{s0}),$$

$$u = -\bar{\mu}\frac{\partial p}{\partial x}, \quad v = -\bar{\mu}\frac{\partial p}{\partial y},$$

$$d(t) = 8\,(1 - m)\,\varepsilon_{bw}\mathrm{Fo} \sum_{n,m,k=1}^{\infty} \frac{\mu_{nmk}^2 e^{-\mu_{nmk}^2 \mathrm{Fo}t}}{\mu_n^2\mu_m^2\mu_k^2} + \mathrm{F}t^{-0.5},$$

$$c(t) = m + (1 - m)\,\varepsilon_{bw}\left(1 - 8\sum_{n,m,k=1}^{\infty} \frac{e^{-\mu_{nmk}^2\mathrm{Fo}t}(1 + \mu_{nmk}^2\mathrm{Fo}t)}{\mu_n^2\mu_m^2\mu_k^2}\right) + \mathrm{F}t^{0.5},$$

$$\mu_{nmk}^2 = \mu_n^2 + \mu_m^2 (a/b)^2 + \mu_k^2 (a/c)^2$$

(for Fo t > 1 the discussion can be limited to one term of the series in the expressions for c(t) and d(t)).

The variables x and y in (11) are related to the well separation L, the rate components are related to $V = Q/(hL)$, the pressure to $\Delta P = Q\mu_0/(hk_e)$, $\mu_0 = \mu(T_0)$, and the time to $L/V = hL^2/Q$. The dimensionless temperature is $\theta = (T - T_s)/\Delta T$, $\Delta T = T_0 - T_s$.

The dimensionless criteria Fo and F, characterizing the heat exchange between rock slabs and the liquid and between the enclosing massif and the liquid have the form

$$\mathrm{Fo} = \frac{a_b}{(a/2)^2}\frac{hL^2}{Q}, \quad \mathrm{F} = 2\varepsilon_{rw}L\sqrt{\frac{a_r}{\pi Qh}}.$$

The Péclet number $\mathrm{Pe} = Q/(a_w h)$, $\varepsilon_{bw} = c_b/c_w$ and $\varepsilon_{rw} = c_r/c_w$ (c_b are the volume heat capabilities for rock slabs).

The viscosity ratio is

$$\bar{\mu} = \frac{\mu_0}{\mu(T)} = 10^{248\frac{\Delta T(\theta-1)}{(133+T_0)(133+T_s+\theta\Delta T)}}.$$

The boundary conditions are $\theta(x, y, 0) = 1$, $\theta(x, y, t) = 1$, $\partial p/\partial n = 0$ (at the boundary of the stratum, n is the normal to the boundary), and $\theta(x_{si}, y_{si}, t) = 0$.

As $a \to 0$ (when the size of the rock slabs is vanishingly small), i.e., as $\mathrm{Fo} \to \infty$, we have $c(t) \to m + (1 - m)\varepsilon_{bw} + \mathrm{F}t^{0.5}$ and $d(t) \to \mathrm{F}t^{-0.5}$. In this case the system of equations (11) corresponds to the system of equations of the homogeneous model of a porous stratum [6]. For Fo = 0, i.e, when the heat exchange does not occur between the rock slabs and the liquid, $c(t) = m + \mathrm{F}t^{0.5}$ and $d(t) = \mathrm{F}t^{-0.5}$.

The fracturing of the stratum was determined in accordance with the assumption of a regular arrangement of the rock slabs, using the formula

$$m = 1 - N_x N_y N_z \frac{abc}{hh_x h_y},$$

where h_x and h_y are the stratum dimensions in the xy plane; and $N_x N_y N_z$ is the number of rock slabs in the stratum in the coordinate directions.

Since $(N_z + 1)s + N_z a = h$ and so forth (if s is the fixed thickness of the cracks in the stratum), we have

$$m = 1 - \frac{(1 - s/h)(1 - s/h_x)(1 - s/h_y)}{(1 + s/a)(1 + s/b)(1 + s/c)}.$$

For a thin stratum ($h \ll h_x$, h_y)

$$m = 1 - \frac{1 - s/h}{(1 + s/a)(1 + s/b)(1 + s/c)}.$$

Moreover, if the rock slabs are thin plates ($a \ll b$, c), then

$$m = \frac{1 + a/h}{1 + a/s}.$$

The complete system of criteria for the problem under consideration contains eight parameters (Fo, F, Pe, ε_{bw}, a/h, a/b, a/c, and h/s) as well as dimensionless function $\bar{\mu}(T_0, T_s)$ of two dimensionless parameters. Obtaining the function t_κ of interest to us (the time in which the dimensionless temperature in the development well drops to $\theta = \kappa$) requires a large number of calculations and so the numerical analysis was carried out for a fixed value $\varepsilon_{bw} = 0.4$ (which corresponds to the ratio of volume heat capacities of granite and water) and without taking into account the conductive heat transfer in the stratum (Pe >> 1) for $\kappa = 0.9$. Moreover, we replaced the parameters a/b and a/c by one, $\alpha = a/b = a/c$ and assumed that $T_s = 10°C$

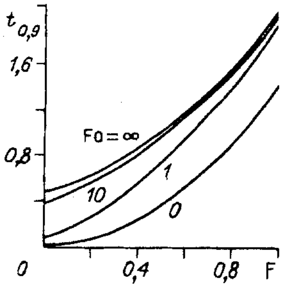

Fig. 1

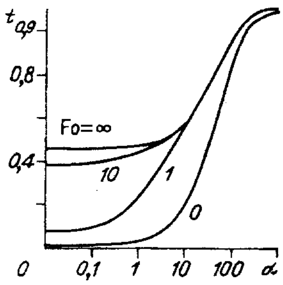

Fig. 2

and T_0 = 50°C. The ratio of the thickness of the stratum to that of the crack h/s = 10^4 and the ratio of the size of the rock slab in the z direction to the stratum thickness is a/h = 0.01. The parameters Fo, F, and α were varied.

The heat balance equation was solved by using an explicit conservative difference scheme without approximation of the convective term by the Ranchel scheme [7] to the first order of accuracy and the pressure equation was solved by the Gauss–Seidel iteration procedure.

The values of the variables at the nodes of the computational net on the flow symmetry axis, i.e., lines joining the wells, were determined from the differnce equations, in which the values of the variables at the nodal points not in the computational region, were eliminated by using the symmetry relations. The temperature at the point where the developmental well was calculated as the weighted mean temperature (over the values of the liquid flow rate) at the four nearest half-nodes of the computational net.

The computations were done on a 51 × 21 net with allowance for the flow symmetry. The computational region was a square with sides of length 3 in dimensionless units, which is entirely sufficient for the boundary conditions at the stratum boundary (they corresponded to the boundary conditions for an infinite stratum) not to affect the characteristics of the flow in the vicinity of the well.

Figure 1 shows $t_{0.9}(F)$ for $\alpha \ll 1$ (the rock slabs are thin plates). The stratum fracturing m, according to (19), is 0.01.

When Fo increases the effective volume heat capacity c(t) of a fractured reservoir increases, causing the heat exchange to slow down. The stratification of the curves according to values of Fo decreases as F increases and when F > 1 the thermal resistance of the rock slabs for Fo > 1 has virtually no effect on the thermal conditions of the stratum, which is determined only by the convective heat transfer and heat exchange with the massif enclosing the stratum.

Figures 2 and 3 show $t_{0.9}(\alpha)$ without allowance for the heat exchange with the enclosing massif for F = 0 and 0.25, respectively.

As α increases from 0 to ∞ the fracturing m of the stratum increases monotonically from the value corresponding to the configuration of rock slabs and extended plates to the value m = 1. The coolant filtering rate decreases and the cooling of the stratum slows down. At the same time as α increases the dimensions of the rock slabs grow, thus speeding up the heat exchange in the stratum.

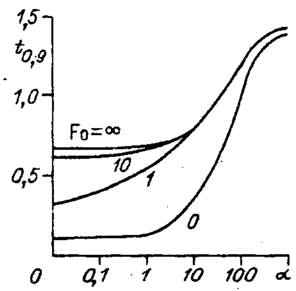

Fig. 3

The second factor, as we see from Figs. 2 and 3, has some effect on the heat exchange in the stratum only in the range $0 < \alpha < 1$. The heat exchange slows down substantially with increasing α when $\alpha > 1$, and when $\alpha > 10$ it is virtually independent of Fo, except for the range Fo < 1, i.e., when the heat exchange between the rock slabs and the liquid is insignificant.

The $t_{0.9}(\alpha)$ curves shown in Figs. 2 and 3 correspond to variable fracturing of the stratum. The function $t_K(\alpha, m)$ decreases monotonically in α and increases monotonically in m, the reason being that with a rigorous regime of coolant extraction, when the flow rates of the injection and developmental wells coincide the coolant filtering rate and, hence, the heat exchange rate on the stratum increases with growing α and decreasing m, since, the latter causes a distribution of the fixed flow rate over a smaller volume of the stratum occupied by the coolant.

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